Implementable Principles of Systems Theory in Educational Tools for Advanced Mathematics

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Aim of Presented Contribution

- An application area – an advanced mathematics: Sets of the differential equations
- A presented innovation – a proposal of a type of an educational tool based on principles of the systems theory
- An implementation environment – a free-available software solution Scicos

Systems Theory > Analyses of Systems

- A physical system can be approximated by a model – with a mathematical description by a **differential equation** (DE) in the following form:

$$a_n y^{(n)}(t) + \ldots + a_1 y'(t) + a_0 y(t) = b_m u^{(m)}(t) + \ldots + b_1 u'(t) + b_0 u(t)$$

- Where:
  - $u(t)$ is an input signal
  - $y(t)$ is an output signal – a solution of DE

Systems Theory > Analyses of Systems

- A multivariable system can be approximated by a model – with a description by **a set of the differential equations**:

$$
\left.\begin{aligned}
&a_{1n}y_1^{(n)}(t)+\ldots+a_{10}y_1(t)+c_{1(n-1)}y_2^{(n-1)}(t)+\ldots+c_{10}y_2(t)=\\
&=b_{1m}u_1^{(m)}(t)+\ldots+b_{10}u_1(t)+d_{1(m-1)}u_2^{(m-1)}(t)+\ldots+d_{10}u_2(t)
\end{aligned}\right\}
$$

$$
\left.\begin{aligned}
&a_{2n}y_2^{(n)}(t)+\ldots+a_{20}y_2(t)+c_{2(n-1)}y_1^{(n-1)}(t)+\ldots+c_{20}y_1(t)=\\
&=b_{2m}u_2^{(m)}(t)+\ldots+b_{20}u_2(t)+d_{2(m-1)}u_1^{(m-1)}(t)+\ldots+d_{20}u_1(t)
\end{aligned}\right\}
$$

Systems Theory > Laplace transformation

- An elementary system - described by DE can be expressed in a suitable form after Laplace transformation of DE to the form of **a transfer function** $G(s)$, which can be further modelled

$$a_n y^{(n)}(t) + \ldots + a_1 y'(t) + a_0 y(t) = b_m u^{(m)}(t) + \ldots + b_1 u'(t) + b_0 u(t)$$

$$a_n Y(s) s^n + \ldots + a_1 Y(s) s + a_0 Y(s) = b_m U(s) s^m + \ldots + b_1 U(s) s + b_0 U(s)$$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_m s^m + \ldots + b_1 s + b_0}{a_n s^n + \ldots + a_1 s + a_0}; m < n$$

Systems Theory > Laplace transformation

- A multivariable system - described by a set of DE's can be expressed in a suitable form after Laplace transformation of a set of DE's to the form of **a matrix of the transfer functions** $\boldsymbol{G}(s)$, which can be further modelled

$$\begin{bmatrix} a_{1n}s^{n}+\ldots+a_{10} & c_{1(n-1)}s^{(n-1)}+\ldots+c_{10} \\ c_{2(n-1)}s^{(n-1)}+\ldots+c_{20} & a_{2n}s^{n}+\ldots+a_{20} \end{bmatrix} \begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} =$$

$$= \begin{bmatrix} b_{1m}s^{m}+\ldots+b_{10} & d_{1(m-1)}s^{(m-1)}+\ldots+d_{10} \\ d_{2(m-1)}s^{(m-1)}+\ldots+d_{20} & b_{2m}s^{m}+\ldots+b_{20} \end{bmatrix} \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix}$$

Systems Theory > Laplace transformation

$$\boldsymbol{G}(s) = \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix}^{-1} \begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix}$$

$$\boldsymbol{G}(s) = \begin{bmatrix} a_{1n}s^{n} + \ldots + a_{10} & c_{1(n-1)}s^{n-1} + \ldots + c_{10} \\ c_{2(n-1)}s^{(n-1)} + \ldots + c_{20} & a_{2n}s^{n} + \ldots + a_{20} \end{bmatrix}^{-1} \cdot$$

$$\cdot \begin{bmatrix} b_{1m}s^{m} + \ldots + b_{10} & d_{1(m-1)}s^{m-1} + \ldots + d_{10} \\ d_{2(m-1)}s^{(m-1)} + \ldots + d_{20} & b_{2m}s^{m} + \ldots + b_{20} \end{bmatrix}$$

$$\boldsymbol{G}(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix}$$

Systems Theory > Modelling Possibilities

$$\boldsymbol{G}(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix}$$

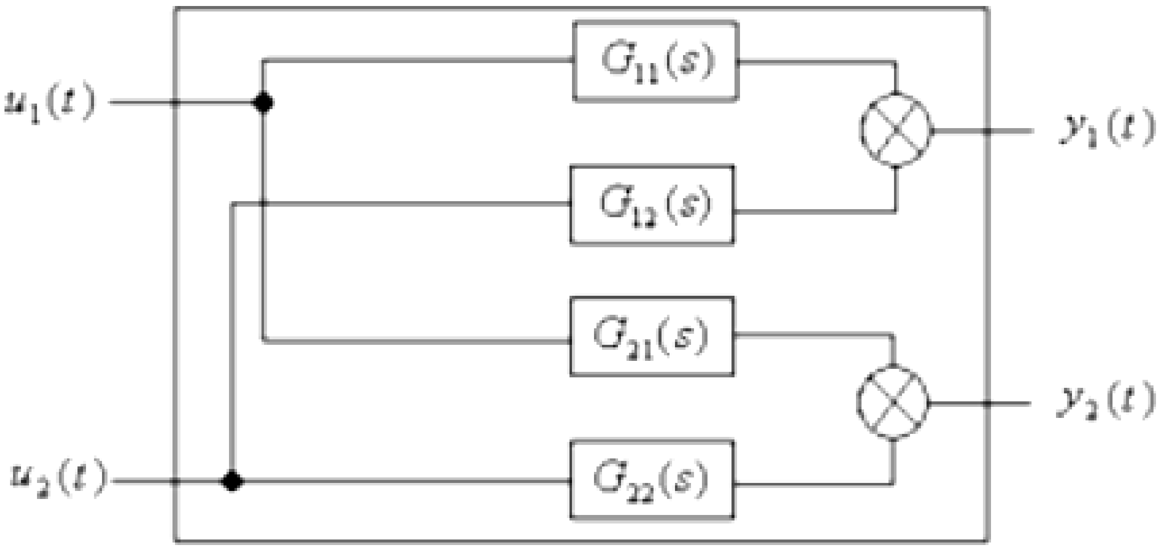

Fig. 1 Modelling Possibilities of Set of DE's in Block Form

Scicos – Suitable Modelling Tool

- A part of a free-available-software environment Scilab
  - A command-line-user interface
  - The practical applications in mathematics, physics, modelling
- Scicos ~ a suitable tool for a circuit modelling of systems
  - Palettes with a wide spectrum of tools
  - Representation of systems with signal rooting, graphs

Scicos Modelling Possibilities

- An elementary system, described by a transfer function, can be modelled by a block defined as *Continuous Transfer Function* (using a definition of numerator and denominator of $G(s)$):

$$G(s) = \frac{b_m s^m + \ldots + b_1 s + b_0}{a_n s^n + \ldots + a_1 s + a_0}$$

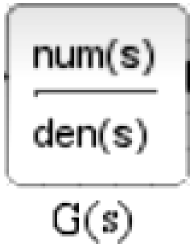

Fig. 2 Block Defined as Continuous Transfer Function in Scicos

Scicos Modelling Possibilities

- A multivariable system, described by a matrix transfer function, can be modelled by a scheme of $\boldsymbol{G}(s)$) – corresponding to Fig. 1

$$\boldsymbol{G}(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix}$$

Scicos Modelling Possibilities

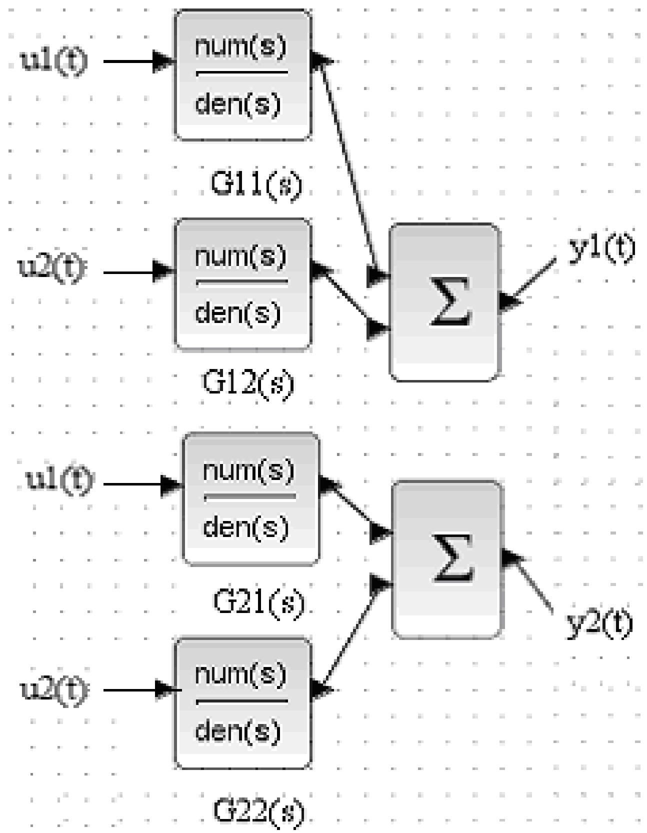

Fig.3 Modelling of Multivariable System in Scicos

Results > Educational Example Based on Principles of Systems Theory

The concrete example in the educational process in the advanced mathematics:

$$\left.\begin{aligned} 3y_1' + 2y_1 - y_2 &= u_1; u_1 = t \\ 2y_2' + y_2 - y_1 &= u_2; u_2 = 5e^{-t} \end{aligned}\right\}; y_2(0) = y_1(0) = 0$$

Results > Educational Example Based on Principles of Systems Theory

Creating of an educational scheme by teacher – after modification of problem in a form after Laplace transformation:

$$\left.\begin{array}{l} 3Y_1(s)s + 2Y_1(s) - Y_2(s) = U_1(s) \\ 2Y_2(s)s + Y_2(s) - Y_1(s) = U_2(s) \end{array}\right\}$$

$$\begin{bmatrix} 3s+2 & -1 \\ -1 & 2s+1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \underbrace{\begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix}^{-1} \begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix}}_{\boldsymbol{G}(s)}$$

Results > Educational Example Based on Principles of Systems Theory

$$\boldsymbol{G}(s) = \frac{1}{6s^2 + 7s + 1}\begin{bmatrix} 2s+1 & 1 \\ 1 & 3s+2 \end{bmatrix}$$

$$\boldsymbol{G}(s) = \underbrace{\begin{bmatrix} \dfrac{2s+1}{6s^2+7s+1} & \dfrac{1}{6s^2+7s+1} \\ \dfrac{1}{6s^2+7s+1} & \dfrac{3s+2}{6s^2+7s+1} \end{bmatrix}}_{\begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix}}$$

Results > Educational Example Based on Principles of Systems Theory

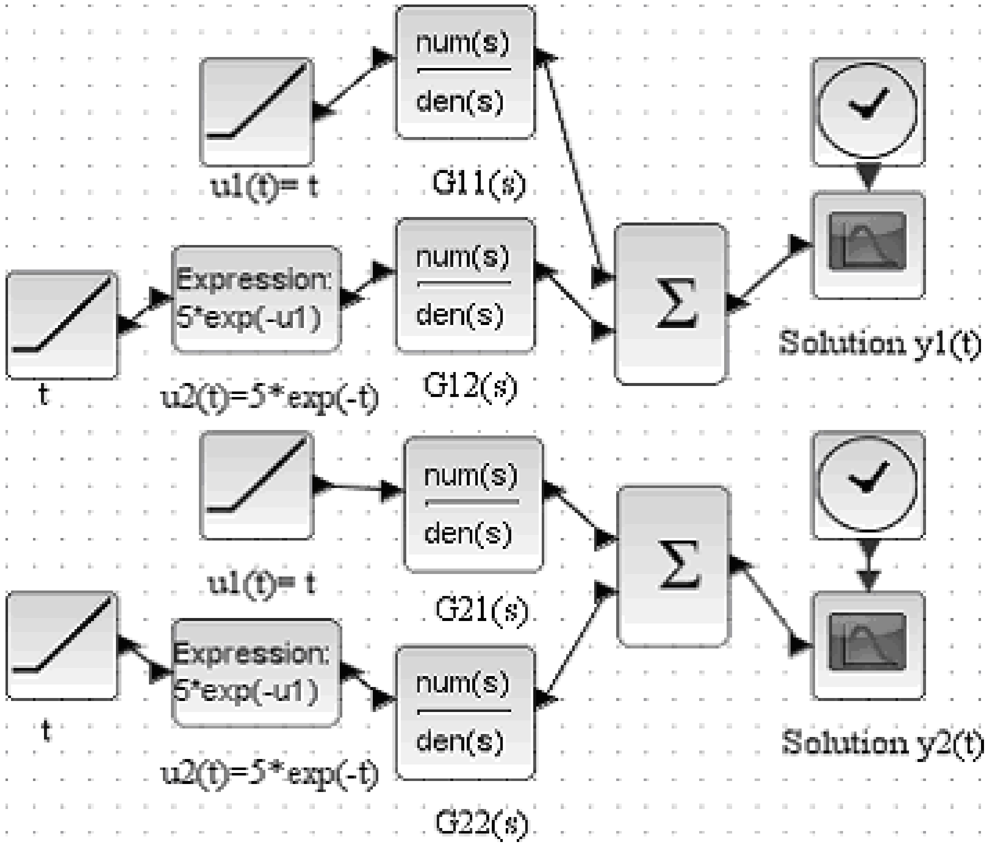

Fig.4 Modelled Problem of Concrete Set of DE’s

Results > Verification

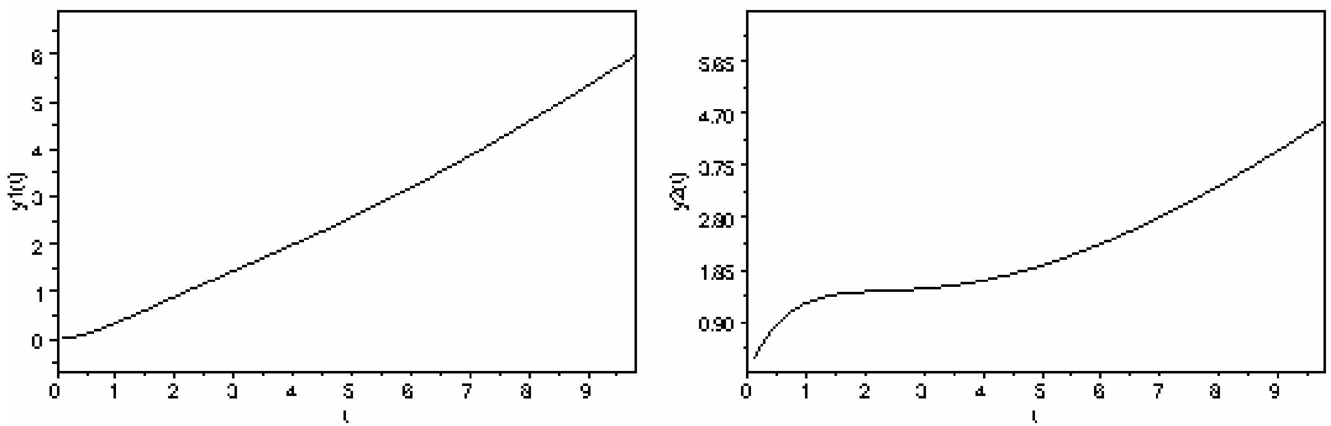

Fig.5 Modelled Solution of Concrete Example in Scicos

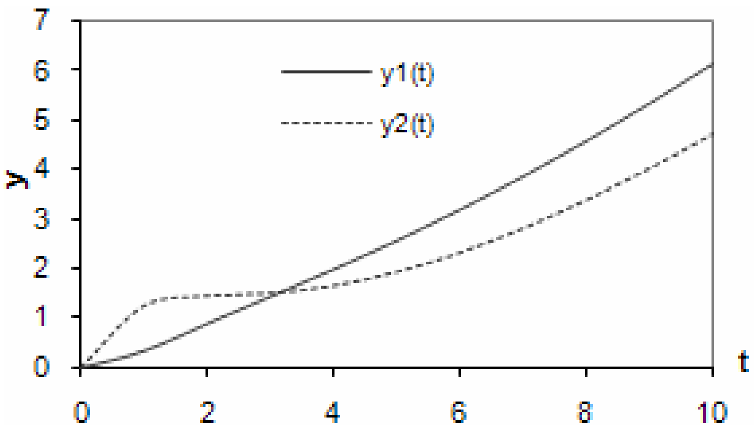

Fig.6 Analytically Solved Concrete Example

$$\left.\begin{aligned} y_1 &= -(1+t).e^{-t} + 6.e^{-\frac{t}{6}} + t - 5 \\ y_2 &= (t-2).e^{-t} + 9.e^{-\frac{t}{6}} + t - 7 \end{aligned}\right\}$$

Conclusions

- The type of educational tool was proposed on principles of the systems theory in favor of:
  - Verifications of examples of sets of ODE by students
  - The modelling practical based examples for students
- This type of the educational tool is not widely described in the advanced mathematics
- The suitable software environment – Scicos
- The systems theory brings new possibilities for proposing of the educational tools in generall

Thank You for Your Attention

Please, do not hesitate to ask,

if you would have any questions.

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